Title: (Instantaneous) x-acceleration

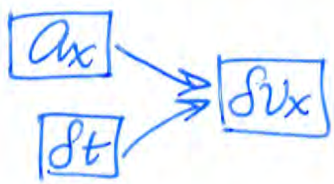
Ingredients

Sketch

At/Through	$[t_i, t_f]$
Owner	System; Frame
Quantity	Tiny change in x-velocity; Instantaneous x-acceleration; Tiny elapsed duration
Variable	δv_x; a_x; δt
Giver	

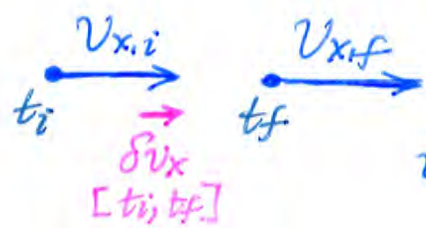
Recipe

Diagram the relationship

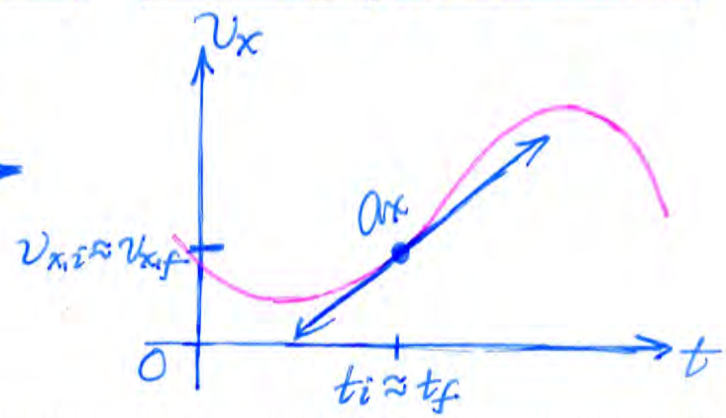


Graphically present quantities

Tiny velocity-change vector



On v_x–t plot: Slope of tangent line



Mathematical relationship

$$a_x \, \delta t = \delta v_x$$

$$a_x = \frac{\delta v_x}{\delta t}$$

Recipe number **K6**: The **title** of this recipe sheet is **"(Instantaneous) x-acceleration**", with the word "Instantaneous" surrounded by parentheses to indicate this word is often omitted.

The top half of this sheet consists of an "**Ingredients**" section with a row labeled "Sketch", a row labeled "At/Through", a row labeled "Owner", a row labeled "Quantity", a row labeled "Variable", and a row labeled "Giver." In this sheet, the row labeled "Giver" isn't used.

For the "Sketch", draw two mostly overlapping snapshots showing a cart moving toward the right across a firm surface. Indicate that the snapshots correspond to almost identical moments in time by drawing the second snapshot of the cart in almost the same position as in the first, with the first snapshot almost completely covering the second snapshot. Draw trailing motion-blur streaks or so-called "whooshies" to emphasize instantaneous motion. Draw a dashed bubble around the earlier snapshot of the cart, at the left, to indicate that the cart is the so-called "System". Draw an arrow labeled +x to indicate that the positive-x direction points to the right.

In the rows of the "Ingredients" section other than the row for the sketch, document the following relationships, using flowchart paths, if helpful: There are two "Owners": one is the "System", and the other is the "Frame". For the interval from initial time t_i (t-sub-i) to final time t_f (t-sub-f), the system has both the "Quantity" called "Tiny change in x-velocity" denoted by the "Variable" (lowercase-delta v-sub-x) and the "Quantity" called "Instantaneous x-acceleration" denoted by the "Variable" a_x (a-sub-x). Also for the same interval from initial time t_i (t-sub-i) to final time t_f (t-sub-f), the "Frame", meaning the collection of rulers and clocks used to make measurements and referred together as the "frame of reference", has the "Quantity" called "Tiny elapsed duration" denoted by the "Variable" (lowercase-delta t).

The bottom half of this sheet consists of a "**Recipe**" section with a row labeled "Diagram the relationship", a row labeled "Graphically present quantities", and a row labeled "Mathematical relationship".

In the row labeled, "Diagram the relationship", draw a flowchart arrow showing that instantaenous x-acceleration a_x (a-sub-x) contributes to the tiny change in x-velocity (lowercase-delta v-sub-x). Draw another arrow showing that tiny elapsed duration (lowercase-delta t) also contributes to the tiny change in x-velocity (lowercase-delta v-sub-x). Recite a story: "Greater instantaneous x-acceleration through a given brief elapsed duration results in a greater tiny change in x-velocity, but even if the instantaneous x-acceleration remained unchanged, simply extending the tiny elapsed duration would also result in a greater tiny change in x-velocity."

The row labeled "Graphically present quantities" will be used for two sections.

For the first section, write the title "Tiny velocity-change vector". Draw two dots from a breadcrumb motion diagram representing snapshot locations of the cart in the "Sketch". Leave some space between the dots. Underneath the first dot (the one at the left), write the label t_i (t-sub-i), and underneath the second dot (the one at the right), write the label t_f (t-sub-f). Draw an initial velocity vector with its tail attached to the first dot (the one at the left) and its head pointing toward the right. Label this velocity vector with the initial x-velocity $v_{x,i}$ (v-sub-x-i). Draw a final velocity vector with its tail attached to the second dot (the one at the right) and its head pointing toward the right. Make this velocity vector very slightly longer than the initial velocity vector. Label this velocity vector with the final x-velocity $v_{x,f}$ (v-sub-x-f). Between the initial velocity vector and the second dot (or nearby, below, if there isn't much room for drawing), draw a small arrow pointing to the right. Label this tiny change-in-velocity vector as the tiny change in x-velocity (lowercase-delta v-sub-x), and also label this change-in-velocity vector with the time interval $[t_i, t_f]$ (open-bracket t-sub-i comma t-sub-f close-bracket). Scale the tiny change-in-velocity vector so that when the initial velocity vector and the tiny change-in-velocity vector are stacked together head-to-tail, their combined size and direction match the size and direction of the final velocity vector.

For the second section, write the title "On (v-sub-x-t) plot: Slope of tangent line". Create an axis system with x-velocity v_x (v-sub-x) on the vertical axis and time t on the horizontal axis. Draw a smooth plot with some variety of v_x (v-sub-x) values (the exact shape isn't very important). Draw a single dot on the plot. Draw the corresponding tickmark on the t axis and label this tickmark (t-sub-i approximately equal to t-sub-f). Draw the dot's corresponding tickmark on the v_x (v-sub-x) axis and label this tickmark (v-sub-x-i approximately equals v-sub-x-f). Draw a tangent line to the plot through the dot. Using slanted writing, label the tangent line a_x (a-sub-x).

In the row labeled, “Mathematical relationship”, write (a-sub-x times lowercase-delta t = lowercase-delta v-sub-x) and (a-sub-x equals lowercase-delta v-sub-x divided by lowercase-delta t).